В. №5.

№1.

а) $\sum_{n=1}^{\infty} (-1)^n (n^2+2) \ln \frac{n^2+1}{n^2}$

$$\lim_{n\to\infty} (n^2+2) \ln \frac{n^2+1}{n^2} = \lim_{n\to\infty} \frac{(n^2+2)}{n^2} =$$

$$= \lim_{n\to\infty} \left(1+\frac{2}{n^2}\right) = 1 \Rightarrow \text{расх.}$$ +

б) $\sum_{n=1}^{\infty} 2^n \left(\frac{n}{n+1}\right)^{n^2}$

$$\lim_{n\to\infty} \sqrt[n]{2^n \left(\frac{n}{n+1}\right)^{n^2}} = \lim_{n\to\infty} 2\left(\frac{n}{n+1}\right)^n =$$

$$= \lim_{n\to\infty} 2\left(\left(1-\frac{1}{n+1}\right)^{-(n+1)}\right)^{\frac{n}{-(n+1)}} = \lim_{n\to\infty} 2\cdot e^{-\frac{n}{n+1}} =$$

$$= \frac{2}{e} < 1 \Rightarrow \text{сх.}$$ +

№2. +

$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^n}{(2n+1)^n}$

$$\lim_{n\to\infty} \sqrt[n]{\frac{n^n}{(2n+1)^n}} = \lim_{n\to\infty} \frac{n}{2n+1} = \frac{1}{2} < 1 \Rightarrow \text{абс. сход.}$$

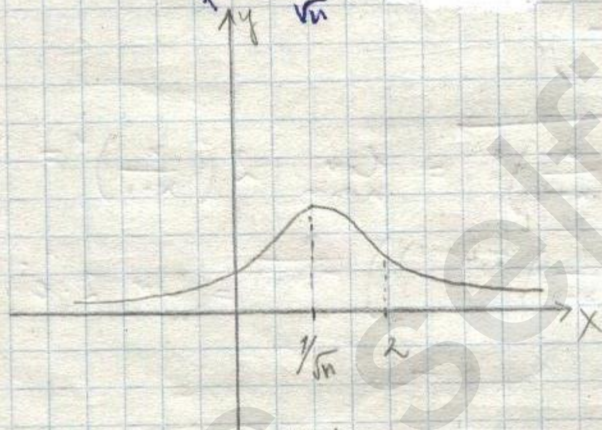
№5.

$$\sum_{n=1}^{\infty} e^{-(1-x\sqrt{n})^2} \qquad [1;2]$$

$$\left(e^{-(1-x\sqrt{n})^2}\right)' = e^{-(1-x\sqrt{n})^2}\left(2(1-x\sqrt{n})\cdot\sqrt{n}\right)=0$$

$$2\sqrt{n}(1-x\sqrt{n})=0$$

$$x=\frac{1}{\sqrt{n}}$$



$$M:\ \sum_{n=1}^{\infty} e^{-(1-\sqrt{n})^2}$$

$$\int_1^{\infty} e^{-(1-\sqrt{y})^2}\,dy$$ – сходится (интеграл Пуассона)

$\Downarrow$

числ. р. сх-ся ; М – сх-ся $\Rightarrow$ равномерно сх-ся на [1; 2]

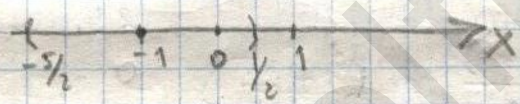
№3.

$$\sum_{n=2}^{\infty} \left(\frac{2}{3}\right)^n \sin\frac{\pi}{n} (x+1)^n$$

1) $$\lim_{x\to\infty} \frac{\cancel{2^{n+1}}^{\,2} \left|\sin\frac{\pi}{n+1}\right| \cancel{|x+1|^{(n+1)}}^{\,|x+1|} \cancel{3^n}}{\cancel{3^{n+1}}_{\,3} \left|\sin\frac{\pi}{n}\right| |x+1|^n \cdot \cancel{2^n}} =$$

$$= \lim_{x\to\infty} \frac{2\,|x+1| \left|\frac{\pi}{n+1}\right|}{3\cdot\left|\frac{\pi}{n}\right|} = \frac{2}{3}|x+1| < 1$$

$R = 3/2$



2) $x = \frac{1}{2}$

$$\sum_{n=2}^{\infty} \left(\frac{2}{3}\right)^n \sin\frac{\pi}{n} \left(\frac{3}{2}\right)^n = \sin\frac{\pi}{n} \sim \frac{\pi}{n} \text{ — расх.}$$

$x = -\frac{5}{2}$

$$\sum_{n=2}^{\infty} \left(\frac{2}{3}\right)^n \sin\frac{\pi}{n} (-1)^n \left(\frac{3}{2}\right)^n = (-1)^n \sin\frac{\pi}{n} \sim$$

$$\sim (-1)^n \frac{\pi}{n} \text{ — сх. условно}$$

3) обл. сх-ти: $\left[-\frac{5}{2}; \frac{1}{2}\right)$

абс. сх-ти: $\left(-\frac{5}{2}; \frac{1}{2}\right)$

обл. равн. сх. $|x+1| \leq r < 3/2$

№4 (+)

а) $\sin(x+2)$ $\qquad x_0 = -1$

$$\sin((x+1)+1) = \sin(x+1)\cos 1 + \cos(x+1)\sin 1 =$$

$$\sin(x+1) = \sum_{n=0}^{\infty} (-1)^n \frac{(x+1)^{2n+1}}{(2n+1)!}$$

$$\cos(x+1) = \sum_{n=0}^{\infty} (-1)^n \frac{(x+1)^{2n}}{(2n)!}$$

$$= \sum_{n=0}^{\infty} (-1)^n \left(\frac{(x+1)^{2n+1}}{(2n+1)!} \cos 1 + \frac{(x+1)^{2n}}{(2n)!} \sin 1 \right) =$$

$$= \sin 1 + \cos 1\,(x+1) - \sin 1 \frac{(x+1)^2}{2!} - \cos 1 \frac{(x+1)^3}{3!} + \ldots$$

$$= \sum_{n=0}^{\infty} \frac{(x+1)^n}{n!} \sin\left(1 + \frac{n\pi}{2}\right)$$ — ряд Тейлора, по теореме единственности

Р. сх-ся при $x \in (-\infty; +\infty)$

$$\sin^{(n)}(x+2)\Big|_{x=-1} = \sin\left(1 + \frac{n\pi}{2}\right)$$

$k - 105$

$$f^{(105)}(-1) = \sin\left(1 + \frac{105\pi}{2}\right)$$

б) $\ln(x^2+5x+6) \qquad x_0 = 0$

$$\ln((x+2)(x+3)) = \ln(x+2) + \ln(x+3) \;=$$

$$\ln(1+t) = \sum_{n=0}^{\infty} (-1)^n \frac{t^{n+1}}{n+1}$$

$$\ln(x+2) = \ln\left(2\left(1+\frac{x}{2}\right)\right) = \ln 2 + \ln\left(1+\frac{x}{2}\right) =$$

$$= \ln 2 + \sum_{n=0}^{\infty} (-1)^n \frac{\left(\frac{x}{2}\right)^{n+1}}{n+1} \quad \text{– сх. при } |x| < 2$$

$$\ln(x+3) = \ln 3 + \ln\left(1+\frac{x}{3}\right) =$$

$$= \ln 3 + \sum_{n=0}^{\infty} (-1)^n \frac{\left(\frac{x}{3}\right)^{n+1}}{n+1} \quad \text{– сх. при } |x| < 3$$

$$\Rightarrow \text{р. сх. при } |x| < 2$$

$$= \ln 6 + \sum_{n=0}^{\infty} (-1)^n \left(\frac{\left(\frac{x}{2}\right)^{n+1} + \left(\frac{x}{3}\right)^{n+1}}{n+1} \right) =$$

$$= \ln 6 + \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} \left[\left(\frac{1}{2}\right)^{n+1} + \left(\frac{1}{3}\right)^{n+1} \right] x^{n+1}$$

ряд Тейлора по теореме единственности

$$a_n = \frac{(-1)^{n-1}}{n} \left[\left(\frac{1}{2}\right)^n + \left(\frac{1}{3}\right)^n \right] = \frac{f^{(n)}(x_0)}{n!}$$

$k = 105$

$$f^{(105)} = \frac{(-1)^{104}}{\cancel{105}} \left[\left(\frac{1}{2}\right)^{105} + \left(\frac{1}{3}\right)^{105} \right] \cdot \cancel{105!}^{\,104!} =$$

$$= \left[\left(\frac{1}{2}\right)^{105} + \left(\frac{1}{3}\right)^{105} \right] \cdot 104!$$

№8.

$$y'' + x^2 y' + 2xy = x^3$$

$y(0) = 0$ с точностью 0,001 на

$y'(0) = 0$ $[0; 1]$

$$y(x) = \sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + \ldots$$

$$y'(x) = \sum_{n=1}^{\infty} n c_n x^{n-1} = c_1 + 2c_2 x + 3c_3 x^2 + \ldots$$

$$y''(x) = \sum_{n=2}^{\infty} n(n-1) c_n x^{n-2} = 2c_2 + 3\cdot 2\cdot c_3 x + \ldots$$

$$\sum_{n=2}^{\infty} n(n-1) c_n x^{n-2} + x^2 \sum_{n=1}^{\infty} n c_n x^{n-1} + 2x \sum_{n=0}^{\infty} c_n x^n = x^3$$

$$\sum_{n=2}^{\infty} n(n-1) c_n x^{n-2} + \sum_{n=1}^{\infty} n c_n x^{n+1} + \sum_{n=0}^{\infty} 2c_n x^{n+1} = x^3$$

$$y(0) = 0 = c_0$$

$$y'(0) = 0 = c_1$$

$$\begin{array}{l|l}
x^0 & 2c_2 = 0 \Rightarrow c_2 = 0 \\
x^1 & 6c_3 + 2c_0 = 0 \Rightarrow c_3 = 0 \\
x^2 & 12c_4 + c_1 + 2c_1 = 0 \Rightarrow c_4 = 0 \\
x^3 & 20c_5 + 2c_2 + 2c_2 - 1 \Rightarrow c_5 = 1/4\cdot 5 \\
x^4 & 30c_6 + 3c_3 + 2c_3 = 0 \Rightarrow c_6 = 0 \\
x^5 & 42c_7 + 4c_4 + 2c_4 = 0 \Rightarrow c_7 = 0 \\
x^6 & 56c_8 + 5c_5 + 2c_5 = 0 \quad c_8 = -7/7\cdot 8\cdot 4\cdot 5 \\
x^7 & c_9 = 0 \\
x^8 & c_{10} = 0 \\
x^9 & 110c_{11} + 8c_8 + 5c_8 = 0 \Rightarrow c_{11} = 13\cdot 7/11\cdot 10\cdot 7\cdot 8\cdot 4\cdot 5 \\
x^{10} & c_{12} = 0 \\
x^{11} & c_{13} = 0 \\
x^{12} & c_{14} = -13\cdot 13\cdot 7/14\cdot 13\cdot 11\cdot 10\cdot 7\cdot 8\cdot 4\cdot 5
\end{array}$$

$$y(x) = \frac{1}{4\cdot 5}x^3 - \frac{7}{7\cdot 8\cdot 4\cdot 5}x^6 + \frac{13\cdot 7}{11\cdot 10\cdot 7\cdot 8\cdot 4\cdot 5}x^9 -$$

$$- \frac{19\cdot 13\cdot 7}{14\cdot 13\cdot 11\cdot 10\cdot 7\cdot 8\cdot 4\cdot 5} + \dots + (-1)^{n+1}\frac{(6(n-1)+1)!!}{(3n+1)!!\,(3n+2)!!}x^{3n}$$

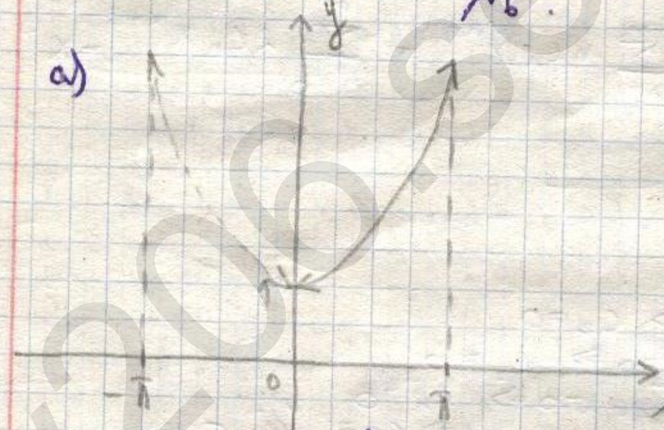
$$|r_n(x)| \le \frac{(6n+1)!!}{(3n+4)!!\,(3n+5)!!}|x|^{3n+3}$$

$$n = 3 \quad y \approx \frac{1}{20}x^3 - \frac{1}{160}x^6 + \frac{13}{17600}x^9$$

$$|r_2(x)| \le \frac{13\cdot 7\cdot 19}{14\cdot 13\cdot 11\cdot 10\cdot 7\cdot 8\cdot 4\cdot 5} < 0{,}001$$

№6.

a)



$y = 2x^2 + 1$

$(0;\pi)$

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\frac{\pi n x}{l}\,dx$$

$b_n = 0$

$$a_0 = \frac{2}{\pi}\int_0^{\pi}(2x^2+1)\,dx = \frac{2}{\pi}\left(\frac{2x^3}{3}+x\right)\Big|_0^{\pi} =$$

$$= \frac{2}{\pi}\left(\frac{2\pi^3}{3}+\pi\right) = \frac{4\pi^2}{3}+2$$

$$a_n = \frac{2}{\pi}\int_0^{\pi}(2x^2+1)\cos nx\,dx \;\circledcirc$$

$2x^2+1 = u$
$4x\,dx = du$
$\cos nx\,dx = dv$
$\frac{1}{n}\sin nx = v$

$$\circledcirc\; \frac{2}{\pi}\left[\frac{1}{n}(2x^2+1)\sin nx\Big|_0^{\pi} - \frac{4}{n}\int_0^{\pi}\sin nx\cdot x\,dx\right] =$$

$$= -\frac{2\cdot 4}{\pi\cdot n}\int_0^{\pi}\sin nx\cdot x\cdot dx \;\circledcirc$$

$x = u$
$dx = du$
$\sin nx\,dx = dv$
$-\frac{1}{n}\cos nx = v$

$$\circledcirc\; -\frac{2\cdot 4}{\pi\cdot n}\left[-\frac{1}{n}x\cos nx\Big|_0^{\pi} + \frac{1}{n}\int_0^{\pi}\cos nx\,dx\right] =$$

$$= -\frac{4\cdot 2}{\pi\cdot n}\left[-\frac{1}{n}\pi(-1)^n + \frac{1}{n^2}\sin nx\Big|_0^{\pi}\right] =$$

$$= -\frac{4\cdot 2}{\pi\cdot n}\left[-\frac{1}{n}\pi(-1)^n\right] = \frac{8}{n^2}(-1)^n$$

$$2x^2+1 \underset{(0,\pi)}{=\!=} \frac{2\pi^2}{3}+1+8\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos nx$$

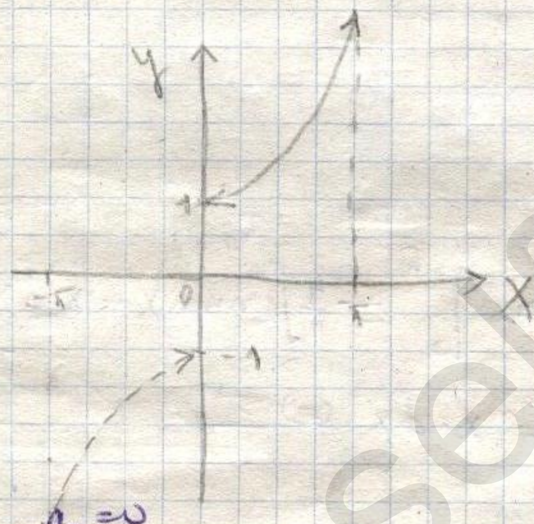
$S_2 = \frac{2\pi^2}{3}+1 - 8\cos x + 2\cos 2x$

$S_3 = \frac{2\pi^2}{3}+1 - 8\cos x + 2\cos 2x - \frac{8}{9}\cos 3x$

$S_{10} = \frac{2\pi^2}{3}+1 - 8\sum_{n=1}^{10}\frac{(-1)^n}{n^2}\cos nx$

$S_1 = \frac{2\pi^2}{3}+1 - 8\cos x$

б)



$a_0 = 0$

$a_n = 0$

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} (2x^2+1)\sin nx\,dx =$$

$2x^2+1 = u$

$4x\,dx = du$

$\sin nx\,dx = dv$

$-\frac{1}{n}\cos nx = v$

$$= \frac{2}{\pi}\left[-\frac{1}{n}\cos nx\cdot(2x^2+1)\Big|_0^{\pi} + \frac{4}{n}\int_0^{\pi}\cos nx\cdot x\,dx\right]$$

$$= \frac{2}{\pi}\left[-\frac{1}{n}(-1)^n(2\pi^2+1)+\frac{1}{n}+\frac{4}{n}\int_0^{\pi}\cos nx\cdot x\,dx\right]$$

$$\int_0^{\pi}\cos nx\cdot x\,dx =$$

$x=u$
$dx=du$
$\cos nx\,dx=dv$
$\frac{1}{n}\sin nx=v$

$$= \frac{1}{n}\sin nx\cdot x\Big|_0^{\pi}-\frac{1}{n}\int_0^{\pi}\sin nx\,dx=$$

$$= \frac{1}{n^2}\cos nx\Big|_0^{\pi}=\frac{1}{n^2}(-1)^n-\frac{1}{n^2}$$

$$b_n=\frac{2}{\pi}\left[-\frac{1}{n}(-1)^n(2\pi^2+1)+\frac{1}{n}+\frac{4}{n}\left(\frac{1}{n^2}(-1)^n-\frac{1}{n^2}\right)\right]=(-1)^n\left(\frac{8}{\pi n^3}-\frac{4\pi}{n}-\frac{2}{n\pi}\right)+\frac{2}{n\pi}-\frac{8}{\pi n^3}=$$

$$2x^2+1\underset{(0,\pi)}{=}\sum_{n=1}^{\infty}\left((-1)^n\left(\frac{8}{\pi n^3}-\frac{4\pi}{n}-\frac{2}{n\pi}\right)+\frac{2}{n\pi}-\frac{8}{\pi n^3}\right)\cdot\sin nx$$

$$S_2=\frac{4\pi^2-12}{\pi}\sin x-2\pi\sin 2x$$

$$S_3=\frac{4\pi^2-12}{\pi}\sin x-2\pi\sin 2x+\frac{12\pi^2-4}{9\pi}\sin 3x$$

$$= \quad S_{10}=\sum_{n=1}^{10}\left((-1)^n\left(\frac{8}{\pi n^3}-\frac{4\pi}{n}-\frac{2}{n\pi}\right)+\frac{2}{n\pi}-\frac{8}{\pi n^3}\right)\sin nx$$

$$= -\frac{1}{n}\int_{-\pi}^{\pi} \sin nx\,dx = \frac{1}{n^2}\cos nx \Big|_{-\pi}^{\pi} = 0$$

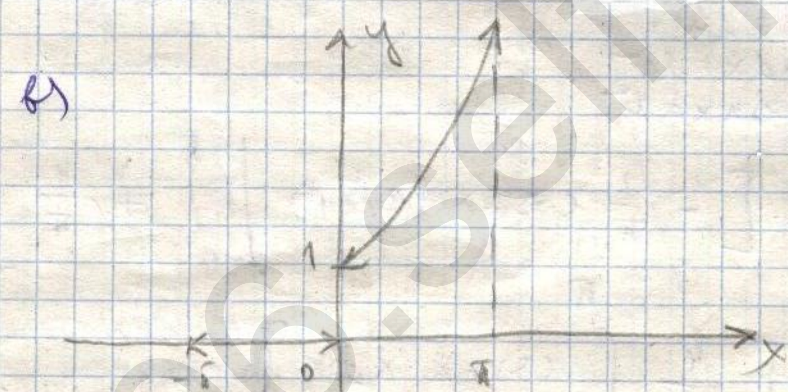
$$2x^2+1 \underset{(0;\pi)}{=} \sum_{n=1}^{\infty} \frac{4}{n^2}(-1)^n \cos nx$$

$$S_2 = -4\cos x + \cos 2x$$

$$S_3 = -4\cos x + \cos 2x - \frac{4}{9}\cos 3x$$

$$S_{10} = \sum_{n=1}^{10} \frac{4}{n^2}(-1)^n \cos nx$$

б)



$$a_n = \frac{1}{\pi}\int_0^{\pi} (2x^2+1)\cos nx\,dx$$

$$\frac{1}{\pi}\int_0^{\pi} (2x^2+1)\cos nx\,dx = \frac{4}{n^2}(-1)^n$$

$$b_n = \frac{1}{\pi}\int_0^{\pi} (2x^2+1)\sin nx\,dx$$

$$\frac{1}{\pi}\int_0^{\pi} (2x^2+1)\sin nx\,dx = (-1)^n\left(\frac{4}{\pi n^3} - \frac{2\pi}{n} - \frac{1}{\pi n}\right) +$$

$$\frac{1}{n\pi} - \frac{4}{\pi n^3}$$

$$2x^2+1 \underset{(0,\pi)}{=} \sum_{n=1}^{\infty}\left[\frac{4}{n^2}(-1)^n\cos nx + \left((-1)^n\left(\frac{4}{\pi n^3} - \frac{2\pi}{n} - \frac{1}{n\pi}\right) + \frac{1}{n\pi} - \frac{4}{\pi n^3}\right)\right]$$

$$S_2 = -4\cos x + \cos 2x + \frac{2\pi^2-6}{\pi}\sin x - \pi\sin 2x$$

$$S_3 = -4\cos x + \cos 2x - \frac{4}{9}\cos 3x + \frac{2\pi^2-6}{\pi}\sin x - \pi\sin 2x + \frac{6\pi^2-2}{9\pi}\sin 3x$$

$$S_{10} = \sum_{n=1}^{10}\left[\frac{4}{n^2}(-1)^n\cos nx + \left((-1)^n\left(\frac{4}{\pi n^3} - \frac{2\pi}{n} - \frac{1}{n\pi}\right) + \frac{1}{n\pi} - \frac{4}{\pi n^3}\right)\sin nx\right]$$

a) $$2x^2+1 \underset{(0,\pi)}{=} \frac{2\pi^2}{3} + 1 + 8\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos nx$$

$$a_0 = \frac{4\pi^2}{3} + 2$$

$$a_n = \frac{8}{n^2}(-1)^n$$

$$\frac{a_0^2}{2} + \sum_{n=1}^{\infty}(a_n^2 + b_n^2) = \frac{1}{l}\int_{-l}^{l} f^2(x)\,dx$$

$$\frac{\left(\frac{4\pi^2}{3}+2\right)^2}{2} + 64\sum_{n=1}^{\infty}\frac{1}{n^4} = \frac{1}{\pi}\int_{-\pi}^{\pi}(2x^2+1)^2\,dx =$$

$= \frac{8}{5}\pi^4 + \frac{8}{3}\pi^2 + 2$

$$\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{1}{64}\left(-\frac{\left(\frac{4\pi^2}{3}+2\right)^2}{2} + \frac{8}{5}\pi^4 + \frac{8}{3}\pi^2 + 2\right) =$$

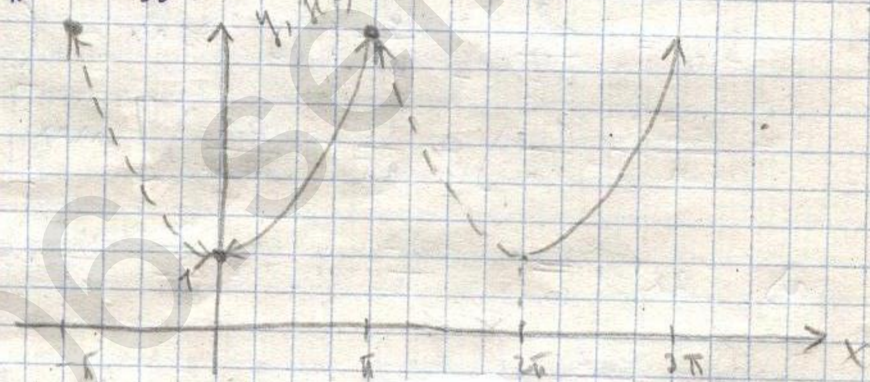
$$= \frac{1}{64}\left[-\frac{16\pi^2}{18} - \frac{16\pi^2}{6} - \cancel{2} + \frac{8}{5}\pi^4 + \frac{8}{3}\pi^2 + \cancel{2}\right] =$$

$$= \frac{1}{64}\left[-\frac{8\pi^4}{9} - \cancel{\frac{8\pi^2}{3}} + \frac{8\pi^4}{5} - \cancel{\frac{8\pi^2}{3}}\right] = \frac{1}{64}\cdot$$

$$\cdot\frac{72\pi^4 - 70\pi^4}{45} = \frac{1}{64}\cdot\frac{32\pi^4}{45} = \frac{\pi^4}{90}$$

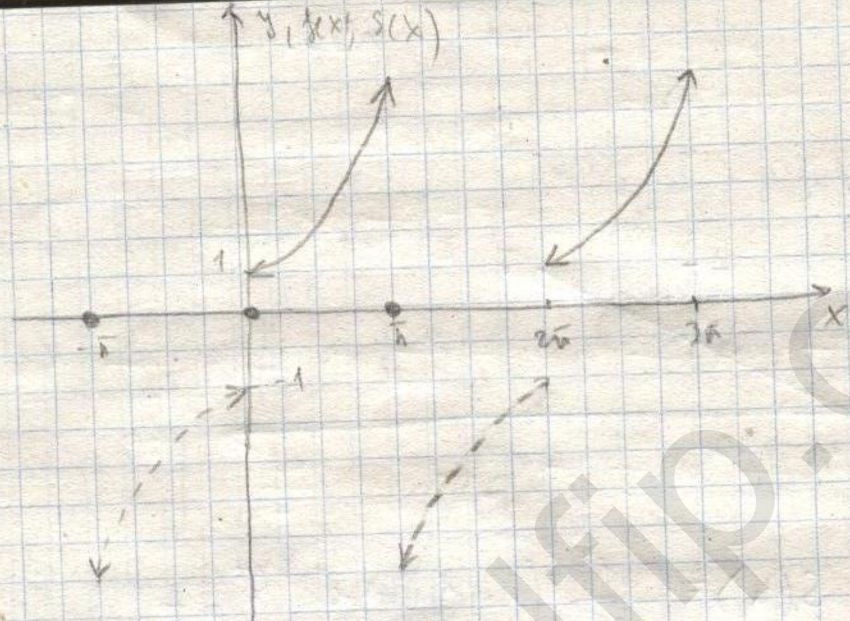
$$\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$$

a)



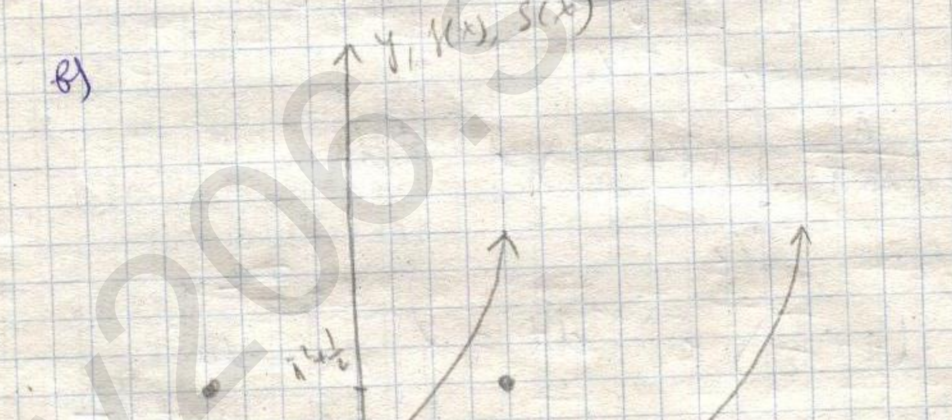
сх.: равномер., поточеч., в среднем

б)



сх.: равн. сх. нет $[-\pi, \pi]$, поточеч. сх. (усл. Дирихле), в среднем

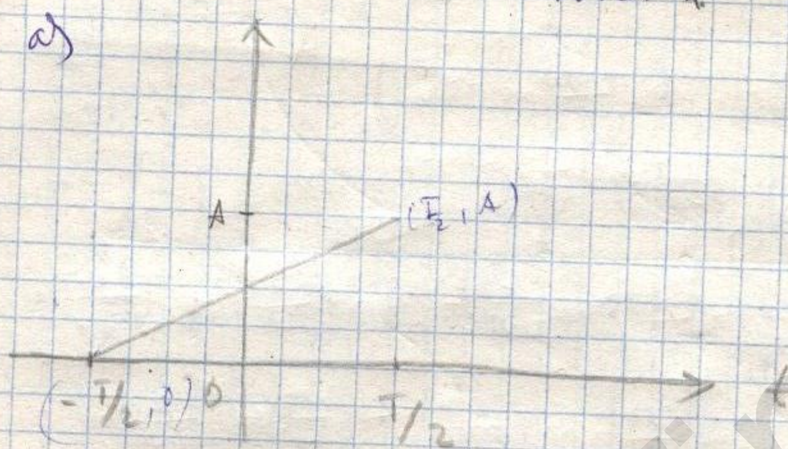
в)



сх.: равн. сх. нет $[-\pi, \pi]$, поточеч. сх., в среднем

N10. 1

a)



$$S(u) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f(t) e^{-iut} dt$$

$$S(u) = \frac{1}{2\pi} \int_{-T/2}^{T/2} \frac{A}{T}\left(t + \frac{T}{2}\right) e^{-iut} dt =$$

$$= \frac{A}{2\pi T} \int_{-T/2}^{T/2} \left(t + \frac{T}{2}\right) e^{-iut} dt$$

$$\int_{-T/2}^{T/2} \left(t + \frac{T}{2}\right) e^{-iut} dt = \left\{ \begin{array}{l} t + \frac{T}{2} = u \\ dt = du \\ dv = e^{-iut} dt \\ v = \frac{e^{-iut}}{-iu} = \frac{i}{u} e^{-iut} \end{array} \right\} =$$

$$= \frac{i}{u}\left(t + \frac{T}{2}\right) e^{-iut} \Big|_{-T/2}^{T/2} - \frac{i}{u} \int_{-T/2}^{T/2} e^{-iut} dt =$$

$$= \frac{i}{u} T e^{-iuT/2} + \frac{1}{u^2} e^{-iut} \Big|_{-T/2}^{T/2} =$$

$$= \frac{i}{u} T e^{-iuT/2} + \frac{1}{u^2} \left(e^{-iuT/2} - e^{iuT/2} \right)$$

$$S(u) = \frac{A}{2\pi T} \left[\frac{i}{u} T e^{-iuT/2} + \frac{1}{u^2} \left(e^{-iuT/2} - \right.\right.$$

$$\left.\left. - e^{iuT/2} \right) \right]$$

$$|S(u)| = \sqrt{Re^2 S(u) + Im^2 S(u)}$$

№7.

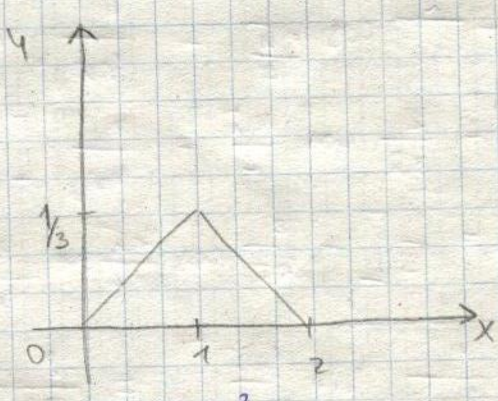
$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

$l = 2$

$u(0,t) = u(2,t) = 0$

$$u(x,0) = \begin{cases} x/3, & 0 \le x \le 1 \\ (2-x)/3 & 1 \le x \le 2 \end{cases}$$

$$\frac{\partial u(x,0)}{dt} = 0$$



$$A_k = \frac{1}{3} \left[\int_0^1 x \sin \frac{k\pi x}{2} dx + \int_1^2 (2-x) \sin \frac{k\pi x}{2} dx \right]$$

$$\int_0^1 x \sin\frac{k\pi x}{2}\,dx = \left\{\begin{aligned} u&=x\\ du&=dx\\ dv&=\sin\frac{k\pi x}{2}\,dx\\ v&=-\frac{2}{k\pi}\cos\frac{k\pi x}{2}\end{aligned}\right\} = -\frac{2\cdot x}{k\pi}\cos\frac{k\pi x}{2}\Big|_0^1 +$$

$$+\frac{2}{k\pi}\int_0^1 \cos\frac{k\pi x}{2}\,dx = -\frac{2}{k\pi}\cos\frac{k\pi}{2} + \frac{4}{(k\pi)^2}\sin\frac{k\pi x}{2}\Big|_0^1 =$$

$$= -\frac{2}{k\pi}\cos\frac{k\pi}{2} + \frac{4}{(k\pi)^2}\sin\frac{k\pi}{2}$$

$$\int_1^2 (2-x)\sin\frac{k\pi x}{2}\,dx = \left\{\begin{aligned} 2-x&=u\\ -dx&=du\\ \sin\frac{k\pi x}{2}\,dx&=dv\\ -\frac{2}{k\pi}\cos\frac{k\pi x}{2}&=v\end{aligned}\right\} =$$

$$= \frac{2(x-2)}{k\pi}\cos\frac{k\pi x}{2}\Big|_1^2 - \frac{2}{k\pi}\int_1^2 \cos\frac{k\pi x}{2}\,dx =$$

$$= \frac{2}{k\pi}\cos\frac{k\pi}{2} - \frac{4}{(k\pi)^2}\sin\frac{k\pi x}{2}\Big|_1^2 =$$

$$= \frac{2}{k\pi}\cos\frac{k\pi}{2} - \frac{4}{(k\pi)^2}\left[\cancel{\sin k\pi}^{\,0} - \sin\frac{k\pi}{2}\right]$$

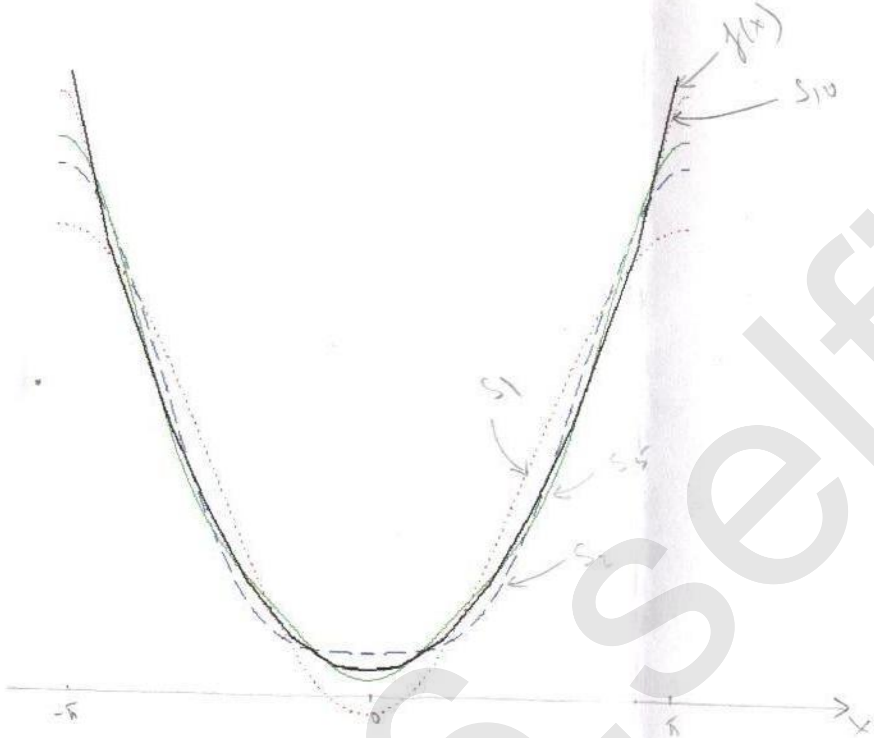
$$A_n = \frac{1}{3}\left[-\cancel{\frac{2}{k\pi}\cos\frac{k\pi}{2}} + \frac{4}{(k\pi)^2}\sin\frac{k\pi}{2} + \cancel{\frac{2}{k\pi}\cos\frac{k\pi}{2}} + \right.$$

$$\left. + \frac{4}{(k\pi)^2}\sin\frac{k\pi}{2}\right] = \frac{8}{3(k\pi)^2}\sin\frac{k\pi}{2} = \begin{cases} 0, & k=2n \\ (-1)^{n+1}\dfrac{8}{3(\pi k)^2}, & k=2n-1 \end{cases}$$

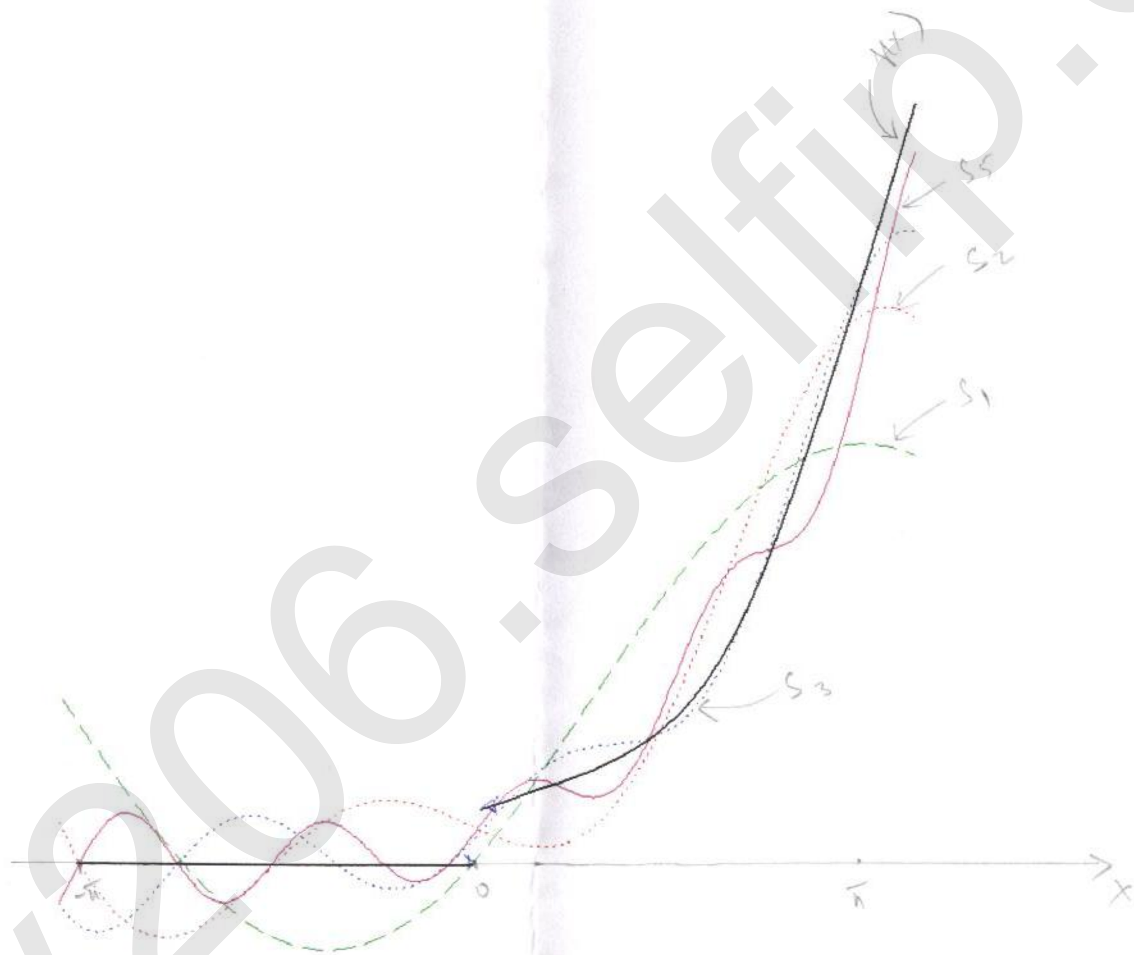
$$u(x,t) = \frac{8}{3\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)^2} \cos \frac{(2n-1)\pi}{2} t \cdot$$

$$\cdot \sin \frac{(2n-1)\pi x}{2}$$

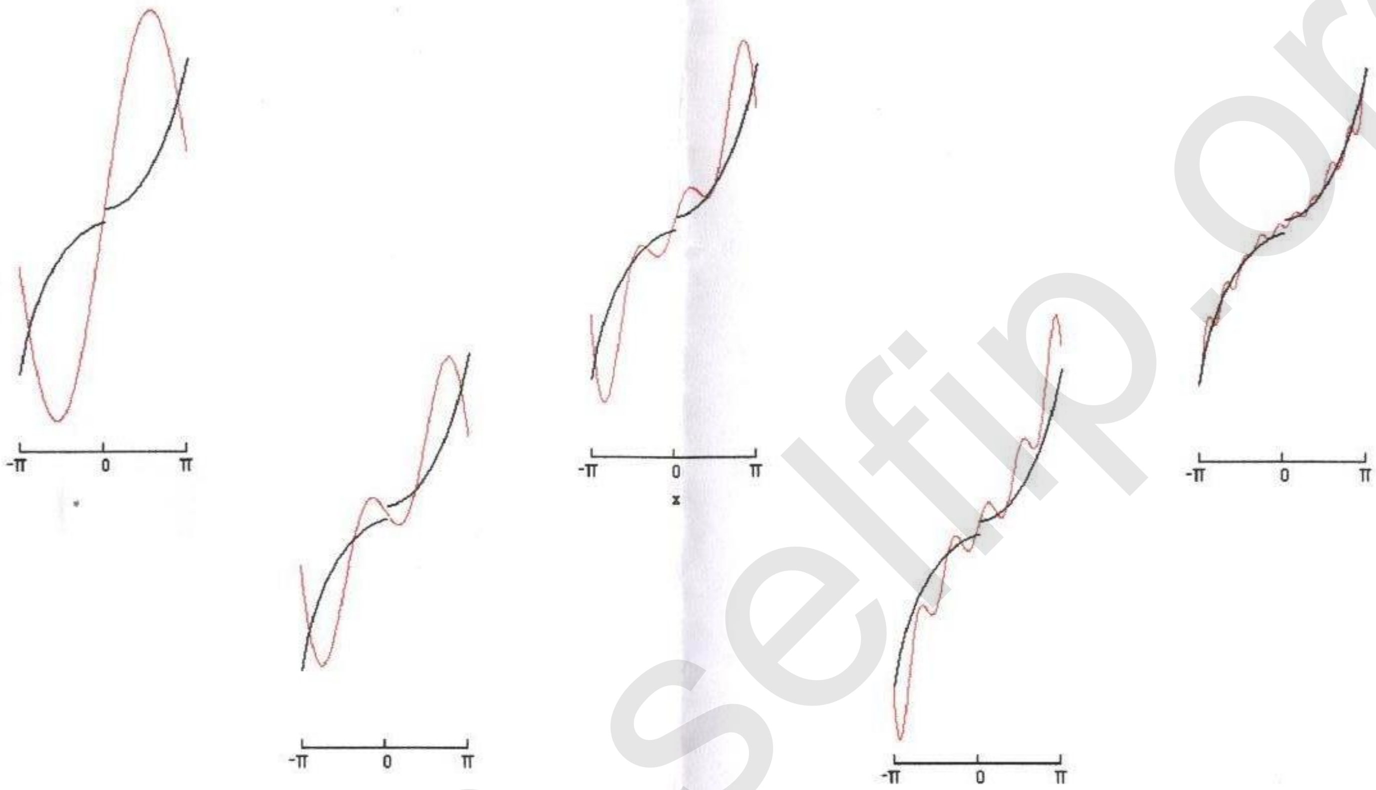
a) $f(x) = 2x^2 + 1$



b)



б)



№3. +

$$\int_0^{0,5} \frac{\operatorname{arctg} x}{x}\, dx$$

$$\frac{\operatorname{arctg} x}{x} = \frac{1}{x}\cdot \operatorname{arctg} x = \frac{1}{x}\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} =$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2n+1}$$

$$\int_0^{0,5} \frac{\operatorname{arctg} x}{x}\, dx = \overset{?}{\sum_{n=0}^{\infty}} \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} \int_0^{0,5} x^{2n}\, dx =$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n\, x^{2n+1}}{(2n+1)(2n+1)} \Bigg|_0^{0,5} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^2\; 2^{2n+1}}$$

$$|r_n(x)| \le \frac{1}{(2n+3)^2\, 2^{2n+3}}$$

$n = 2$.

$$\int_0^{0,5} \frac{\operatorname{arctg} x}{x}\, dx = \frac{1}{2} - \frac{1}{9\cdot 2^3} + \frac{1}{25\cdot 2^5} =$$

$$= \frac{1}{2} - \frac{1}{72} + \frac{1}{800} = 0,4874$$

погрешность $\dfrac{1}{7^2\cdot 2^7} = \dfrac{1}{6272} \approx 0,0001594$

$< 0,001$